Wake Fields and Beam Dynamics

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Overview

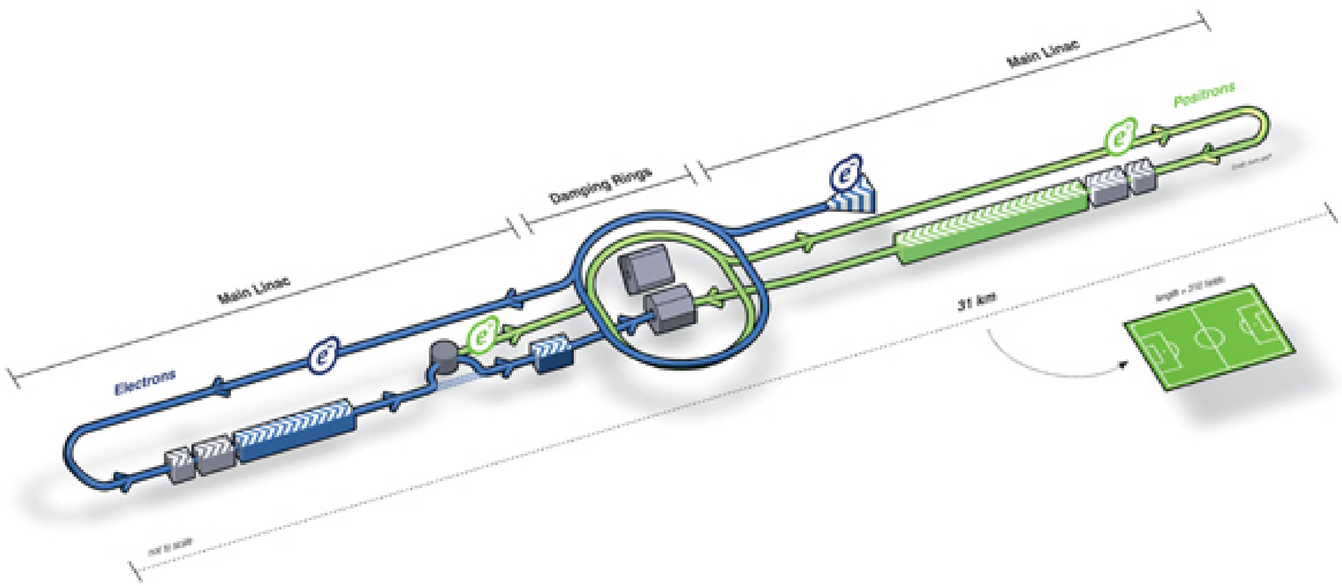
1. A study of how particles affect other particles in accelerators.

    - The importance of wake fields in storage rings.

    - Calculation of particle movements in the presence of wake fields.

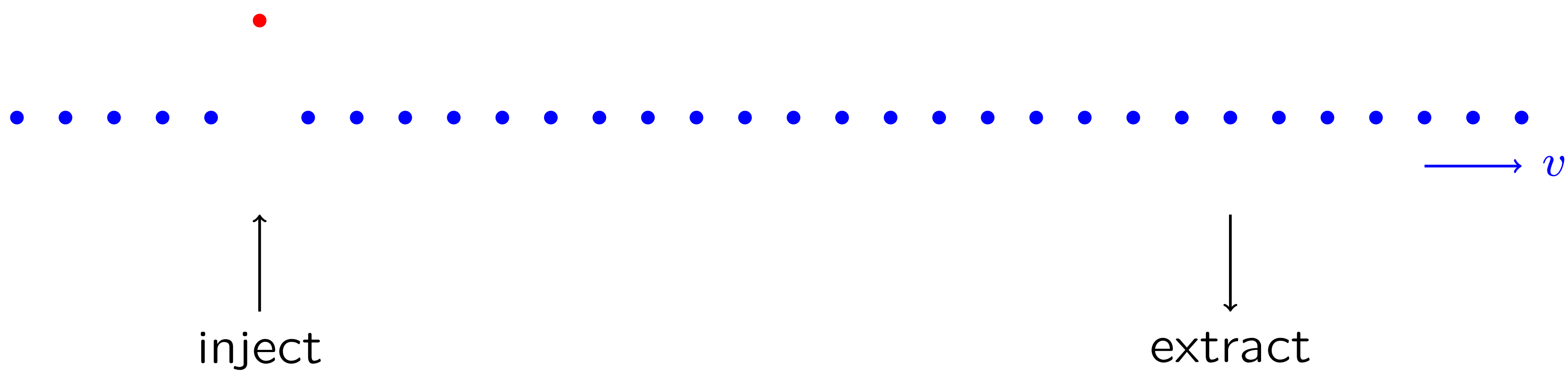
2. Some proposals for future research.

A storage ring is a circular accelerator in which charged particles circulate. The particles lose energy through radiation. As a result, their oscillations are reduced, or damped.

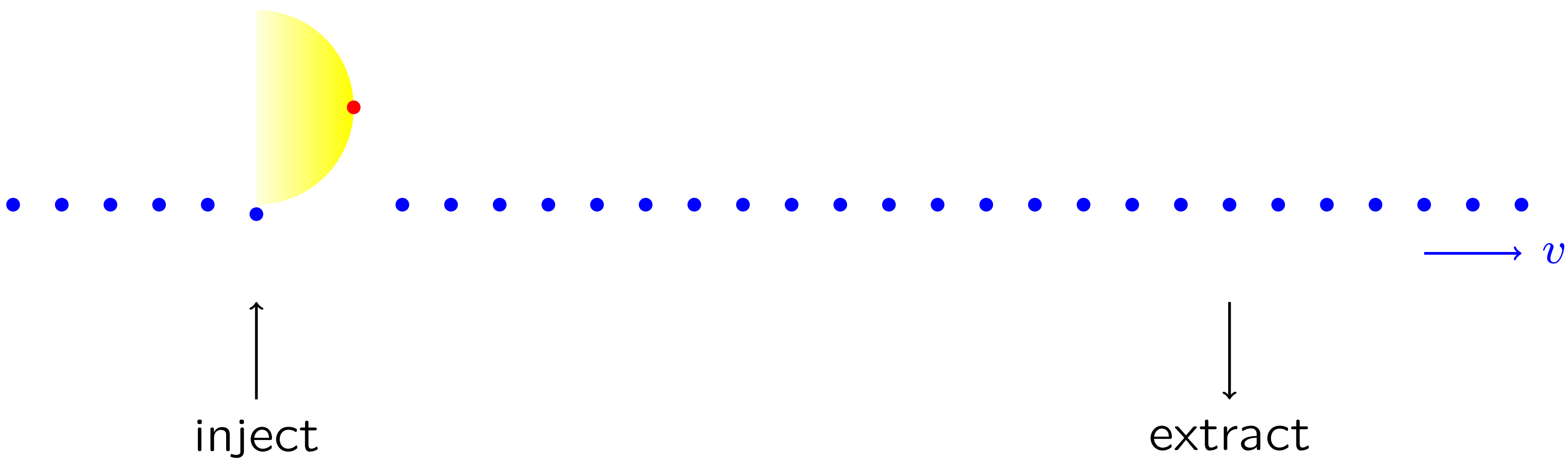


In a linear collider, this effect is used to produce stable, narrow beams. The problem starts when fresh partlces have to be injected before these damped particles are extracted.

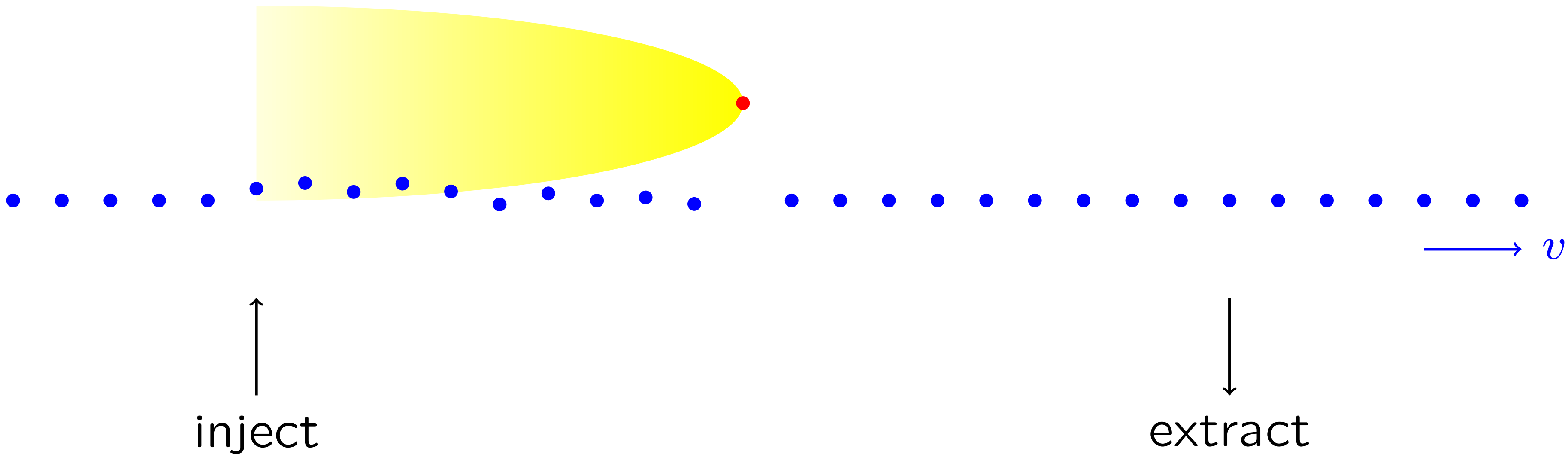
In the ring, a fresh bunch of electrons or positrons is injected.



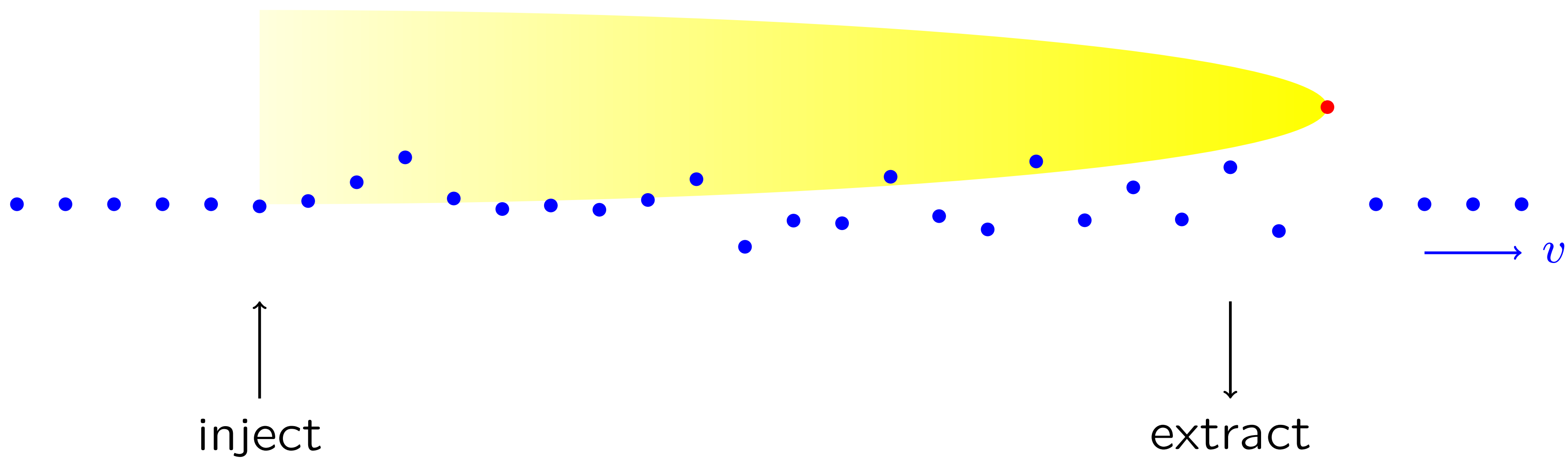
A wake field is generated because of the resistive wall.



This perturbs and induces jitter in the damped bunches ...



... and causes the emittance of the extracted beam to increase.



A bunch of electrons in the ring experiences forces from the focusing magnets, wake fields from leading bunches, and the feedback control system.

The equation of motion for the displacement of the m^{th} bunch is therefore given by

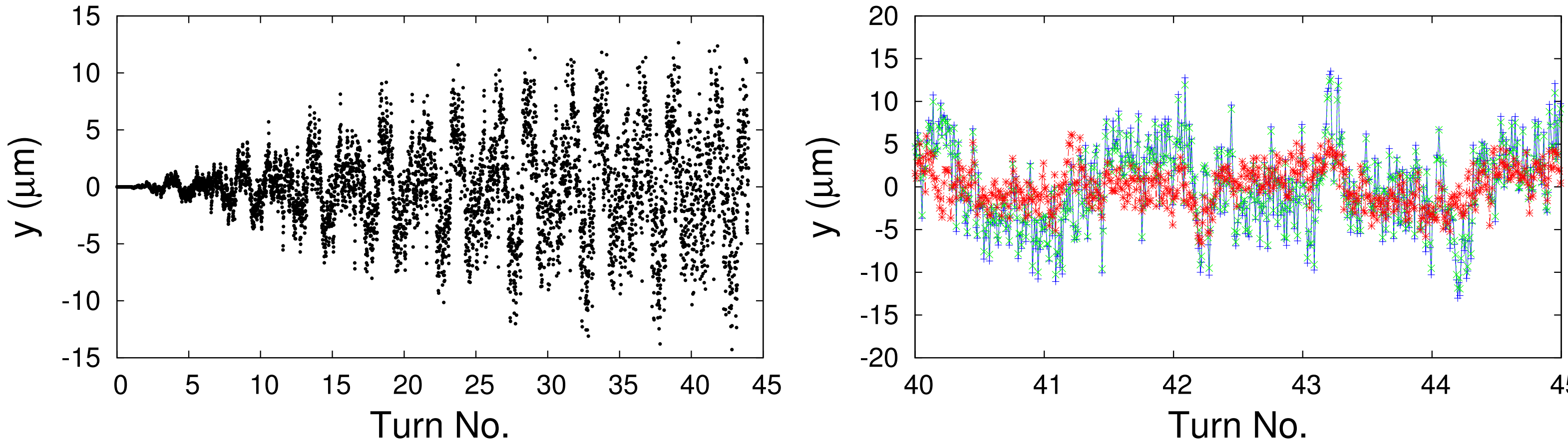
$$\frac{d^2 y_m}{dt^2} + 2\zeta \frac{dy_m}{dt} + K(t) y_m = -\frac{N r_0 c}{\gamma T_0} \sum_{n=1}^{\infty} W_1(-cn\tau) y_{m+n}(t - n\tau)$$

where ζ is the damping factor from feedback control, $K(t)$ is the focusing strength, $W_1(z)$ is the wake function, τ is the for light to traverse the bunch spacing, N the bunch population, T_0 the revolution time, and γ the electron energy.

The simulation can be carried out by numerical integration for any fill pattern of the bunches, as well as injection and extraction procedure.

The simulation is carried out using the lattice for the DCO2 damping ring, and a bunch population of 1.04×10^{10}. The rms of the injection offsets is assumed to correspond to 10% of the recommended betatron action for injected positrons.

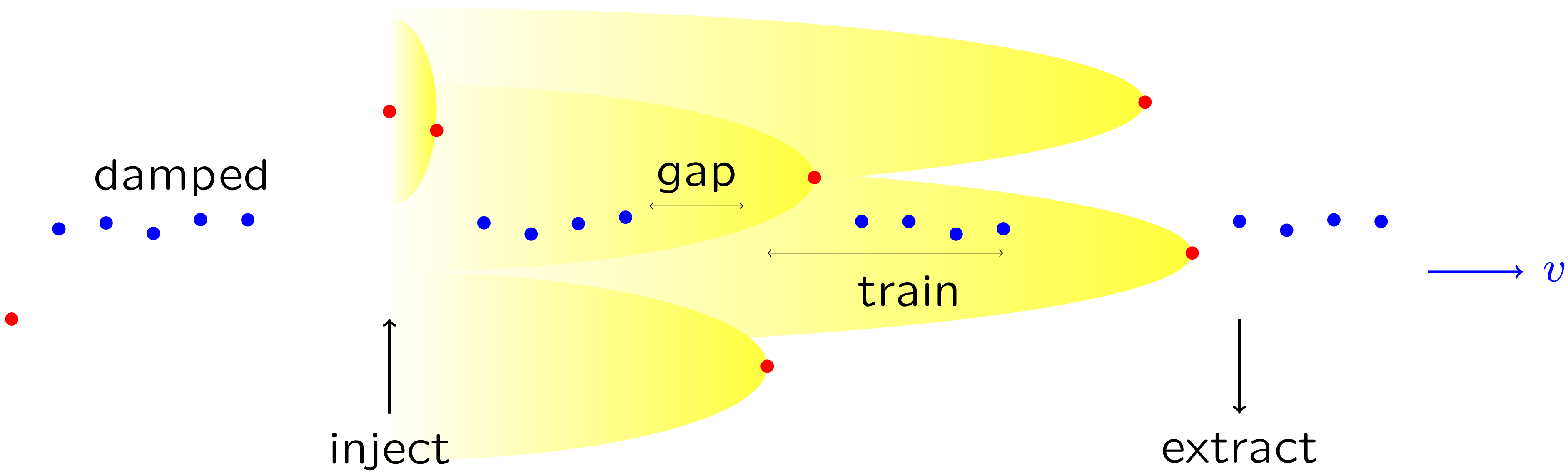
Displacement of each bunch just before extraction is plotted.



The effects of the lattice (red versus blue), the wall of the beam pipe (blue versus green), and different ways of filling the ring with bunches have been studied.

An analytic solution for the jitter.

Injecting over 5000 random bunches and determining the disturbance on the damped bunches is a complex problem. It is therefore surprising that an analyic solution is in fact possible.



The solution relies mainly on three assumptions:

1. The bunches effectively travel at the speed of light.
2. Wake forces among the damped bunches are negligible.
3. Uniform focusing.

Within the turn n_t, the displacement of a damped bunch y_m is:

$$\frac{d^2 y_m}{dt^2} + 2\zeta \frac{dy_m}{dt} + \omega_\beta^2 y_m = Af(n_t T_0) e^{-(t-n_t T_0)/T_f} e^{-i\omega_\beta t}$$

On the left, ζ is the feedback damping factor, ω_β^2 is the averaged focusing strength. On the right, $Af(n_t T_0)$ is wake force at the start of turn n_t, T_f is feedback damping time, and T_0 the revolution time.

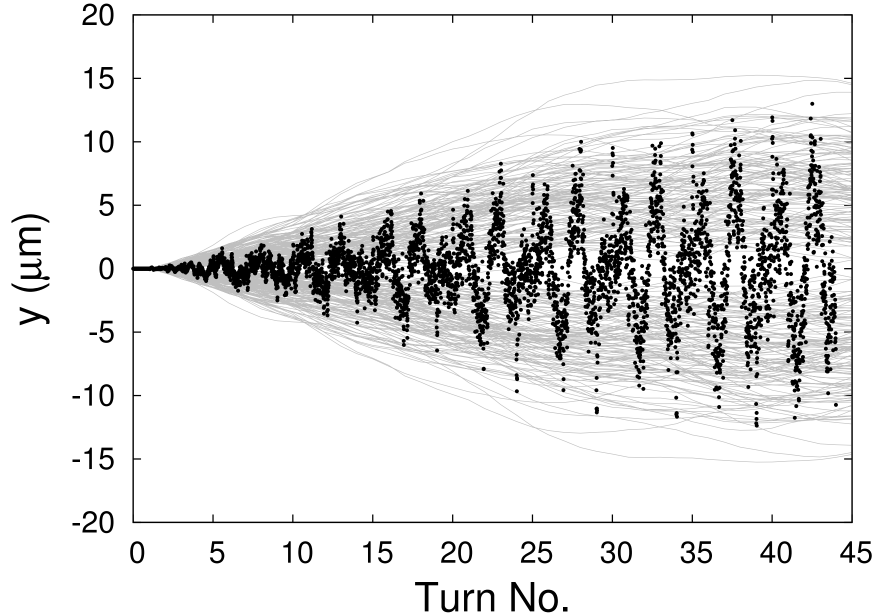
This can be solved analytically, and the solution is given by:

$$y_m(t) = A_{n_t} e^{(-\zeta + i\omega)t} + B_{n_t} e^{(-\zeta - i\omega)t} + p_{n_t} e^{(-\zeta - i\omega_\beta)t} \quad (1)$$

where $p_{n_t} = -a_{n_t} \exp(\zeta n_t T_0)/\zeta^2$ and $\omega = \sqrt{\omega_\beta^2 - \zeta^2}$.

A_{n_t} and B_{n_t} are unknown constants that can be obtained using the condition that the damped bunch has zero displacement and velocity initially.

For a simple estimate, it is sufficient to track only the bunches that are extracted in the final turn. The initial displacements of these bunches are likely to be close to those of the bunches that are extracted earlier.



The grey curves are the amplitudes of the oscillations of these bunches, obtained analytically. The distribution of these curves are very close to the distribution of black dots from simulation.

The study of beam stability under wake fields.

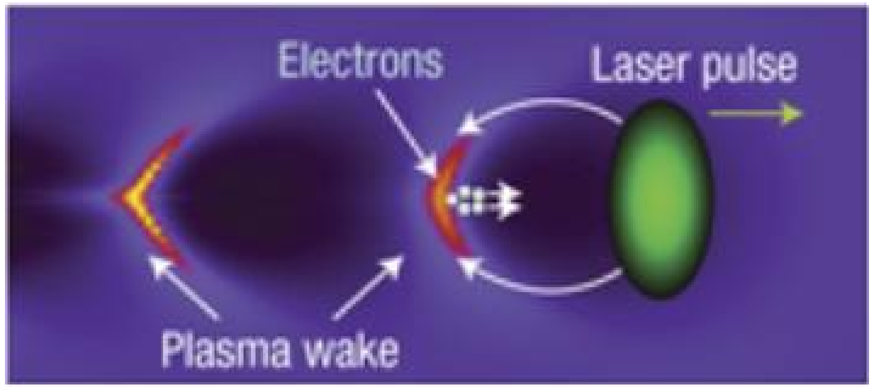
The current line of research is turning out to be interesting in the new physics regimes of accelerators.

There is direct application to future accelerators, such as the International Linear Collider and the Super B Factory.

The research could be extended to study the effect of wake fields on beam dynamics in different parts of the accelerator.

Collaboration with the Cockcroft Institute is fruitful and should continue.

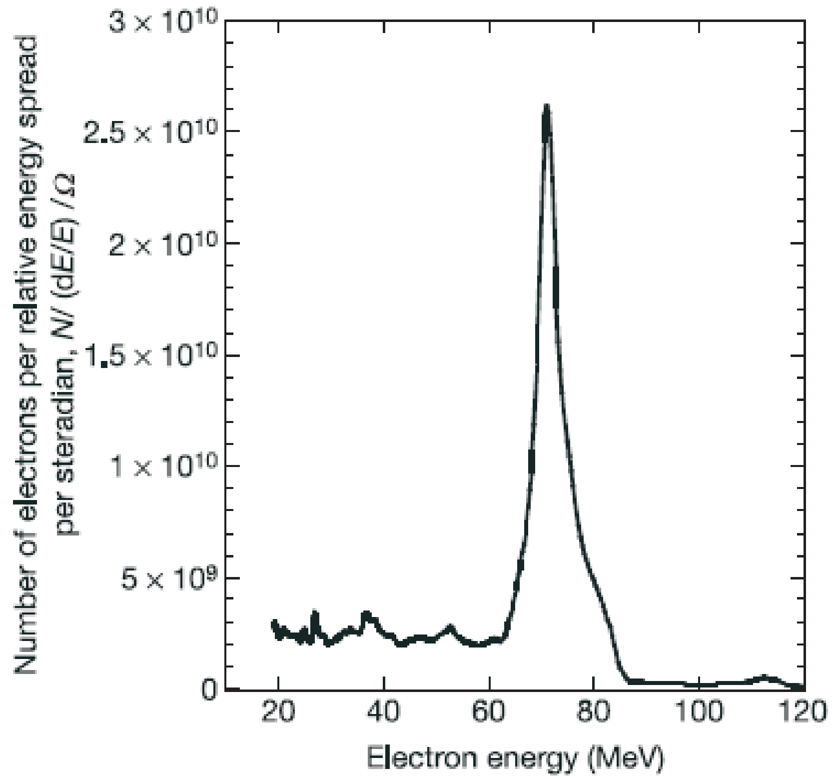
Recent work on acceleration of electrons using wake fields in plasma generated by high power lasers has shown great promise.



Potential applications include not only high energy accelerators, but also for compact light sources which has been demonstrated.

The next breakthrough would be in the technology to control the beam.

Practical problems include not only energy spread of the laser generated electrons, but also the width and guidance of the beam. A major limitation of compact accelerators is the need for bulky magnets.



Many opportunities exist to develop new theories and experiments by merging fluid dynamics and electromagnetics techniques.